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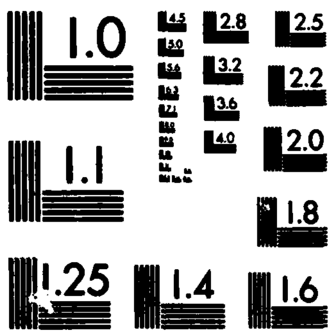

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ACTIVE AND PASSIVE REMOTE SENSING OF ICE

DEPARTMENT OF THE NAVY
OFFICE OF NAVAL RESEARCH
Contract N00014-83-K-0258

SEMI-ANNUAL REPORT

covering the period

February 16, 1983 - July 31, 1983

prepared by

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ACTIVE AND PASSIVE REMOTE SENSING OF ICE

Principal Investigator: Jin Au Kong

Semi-Annual Progress Report

This is a report on the progress that has been made in the study of active and passive remote sensing of ice under the sponsorship of ONR Contract N00014-83-K-0258 during the period of February 16, 1983 - July 31, 1983. During this period we have: (1) derived the dyadic Green's function for a two-layer anisotropic medium; (2) derived the backscattering cross sections and the bistatic scattering coefficients for a two-layer anisotropic random medium; and (3) participated in a planning meeting at the Cold Regions Research and Engineering Laboratory (CRREL) concerning the winter microwave remote sensing measurements.

The dyadic Green's function for a two-layer anisotropic medium has been derived. The anisotropic medium is assumed to be tilted uniaxial. Inside the uniaxial medium there are two characteristic waves, i.e. an ordinary-wave and an extraordinary-wave. An incident wave with either TE or TM polarization produces both TE and TM waves when it is reflected at the interface of an isotropic and a uniaxial anisotropic media because of its coupling into both the o-wave and the e-wave. With the availability of the dyadic Green's function, many electromagnetic wave radiation and scattering problems in layered uniaxial media can be solved. A manuscript has been prepared for submission to a journal for publication [Appendix].

The backscattering cross sections and the bistatic scattering coefficients for a two-layer anisotropic random medium have been derived. The Born approximation is used along with the dyadic Green's function for the two-layer anisotropic medium to calculate the scattered fields. The theoretical results are used to match published experimental data. We are now for the first time able to compute contributions from both the background anisotropy and the anisotropy of the permittivity fluctuations. A manuscript is being prepared to document the theory and the theoretical calculations.

We have participated at the planning meeting at CRREL concerning the winter microwave remote sensing measurements. On June 17, 1983, the Principal Investigator went to a meeting at CRREL concerning the measurement of the physical and electrical properties of fresh water ice and artificially grown sea ice for the determination of the effects of these properties on the emissivity and reflectivity of the ice. A presentation was made to stress the importance of measuring the cross-sectional profile of the ice which has been shown to play a very significant role in the random medium model.

APPENDIX

Dyadic Green's Functions for Layered Anisotropic Medium

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Abstract:

The dyadic Green's functions (DGF) for unbounded and layered anisotropic media have been obtained. The anisotropic medium is assumed to be tilted uniaxial. With the availability of the DGF's, many problems involving radiation and scattering of electromagnetic waves can readily be solved.

I. Introduction

The dyadic Green's functions (DGF) for various configurations have been studied extensively in the literature[1-3]. In applications to microwave remote sensing, the problems of radiation and scattering of electromagnetic waves in layered uniaxial media have presented a fundamental difficulty in the absence of a useful dyadic Green's function. The DGF for a layered uniaxial medium with the optic axes perpendicular to the planes of stratification has been derived[2]. However, in most practical applications, the optic axes of the uniaxial medium is tilted. It is the purpose of this paper to present the results of the DGF for both an unbounded and a layered anisotropic medium corresponding to a uniaxial medium with a tilted optic axis. In order to cast the final results in a simple and interpretable form, the reflection and transmission coefficients at the boundary surface of the anisotropic medium are also obtained.

II. Dyadic Green's Function

In order to facilitate the derivation of the dyadic Green's function (DGF) for an anisotropic-medium layer, we first consider an unbounded anisotropic medium whose permittivity tensor, $\bar{\bar{\varepsilon}}$, is taken to be uniaxial with its optic axis tilted off the z-axis by an angle ψ in the y-z plane as shown in Fig. 1. The permittivity tensors $\bar{\bar{\varepsilon}}^{(0)}$ and $\bar{\bar{\varepsilon}}$, before and after the tilting, respectively, are as follows:

$$\bar{\bar{\varepsilon}}^{(0)} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} . \tag{1}$$

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & \varepsilon_{23} \\ 0 & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \tag{2}$$

where

$$\varepsilon_{11} = \varepsilon, \tag{3a}$$

$$\varepsilon_{22} = \varepsilon \cos^2\psi + \varepsilon_z \sin^2\psi, \tag{3b}$$

$$\varepsilon_{23} = \varepsilon_{32} = (\varepsilon_z - \varepsilon) \cos\psi \sin\psi, \tag{3c}$$

$$\varepsilon_{33} = \varepsilon \sin^2\psi + \varepsilon_z \cos^2\psi. \tag{3d}$$

The DGF for this anisotropic medium, $\bar{\bar{G}}(\bar{r}, \bar{r}')$, satisfies the vector wave equation

$$\nabla \times \nabla \times \bar{\bar{G}}(\bar{r}, \bar{r}') - \omega^2 \mu \bar{\bar{\varepsilon}} \cdot \bar{\bar{G}}(\bar{r}, \bar{r}') = \bar{\bar{I}}\, \delta(\bar{r} - \bar{r}') \tag{4}$$

where ω is the angular frequency and μ is the isotropic permeability of the medium. We write the Fourier transform pair of DGF as follows

$$\bar{\bar{G}}(\bar{r}, \bar{r}') = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} d^3\bar{k}\, \bar{\bar{G}}(\bar{k}, \bar{r}')\, e^{i\bar{k} \cdot \bar{r}} \tag{5}$$

$$\bar{\bar{G}}(\bar{k}, \bar{r}') = \int_{-\infty}^{\infty} d^3\bar{r}\, \bar{\bar{G}}(\bar{r}, \bar{r}')\, e^{-i\bar{k} \cdot \bar{r}}. \tag{6}$$

Substituting (5) into (4) and using the identity

$$\delta(\bar{r} - \bar{r}') = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} d^3\bar{k}\, e^{i\bar{k} \cdot (\bar{r}-\bar{r}')}, \tag{7}$$

we obtain

$$\bar{\bar{G}}(\bar{k}, \bar{r}') = -[\bar{\bar{k}} \cdot \bar{\bar{k}} + \omega^2 \mu \bar{\bar{\varepsilon}}]^{-1}\, e^{-i\bar{k} \cdot \bar{r}'} \tag{8}$$

where

$$\bar{\bar{k}} = \begin{bmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{bmatrix}. \tag{9}$$

Performing the matrix inversion, we obtain

$$\bar{\bar{G}}(\bar{k}, \bar{r}') = \frac{-e^{-i\bar{k} \cdot \bar{r}'}}{\omega^2\mu \; D_1(\bar{k}) \; D_2(\bar{k})} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \tag{10}$$

where

$$D_1(\bar{k}) = k_x^{\,2} + k_y^{\,2} + k_z^{\,2} - \omega^2\mu\varepsilon \tag{11a}$$

$$D_2(\bar{k}) = \varepsilon k_x^{\,2} + \varepsilon_{22}k_y^{\,2} + \varepsilon_{33}k_z^{\,2} + \varepsilon_{23}2k_yk_z - \omega^2\mu\varepsilon\varepsilon_z \tag{11b}$$

$$A_{11} = k_x^{\,2}D_1 + k_x^{\,2}\omega^2\mu(\varepsilon - \varepsilon_z) - \omega^2\mu D_2 \tag{12a}$$

$$A_{12} = A_{21} = k_xk_yD_1 + \omega^2\mu k_x(\varepsilon - \varepsilon_z) \cos\psi \, (k_y \cos\psi - k_z \sin\psi) \tag{12b}$$

$$A_{13} = A_{31} = k_xk_zD_1 + \omega^2\mu k_x(\varepsilon_z - \varepsilon) \sin\psi \, (k_y \cos\psi - k_z \sin\psi) \tag{12c}$$

$$A_{22} = k_y^{\,2}D_1 - \omega^2\mu[\varepsilon k_x^{\,2} + \varepsilon_{33}k_y^{\,2} + \varepsilon_{33}k_z^{\,2} - \omega^3\mu\varepsilon\varepsilon_{33}] \tag{12d}$$

$$A_{23} = A_{32} = k_yk_zD_1 + \omega^2\mu\varepsilon_{23}(k_y^{\,2} + k_z^{\,2} - \omega^2\mu\varepsilon) \tag{12e}$$

$$A_{33} = k_z^{\,2}D_1 - \omega^2\mu[\varepsilon k_x^{\,2} + \varepsilon_{22}k_y^{\,2} + \varepsilon_{22}k_z^{\,2} - \omega^2\mu\varepsilon\varepsilon_{22}] \tag{12f}$$

In the derivation of (11)-(12) we made use of the relationships

$$\varepsilon_{22}\varepsilon_{33} - \varepsilon_{23}^2 = \varepsilon\varepsilon_z \tag{13a}$$

$$\varepsilon_{22} + \varepsilon_{33} = \varepsilon + \varepsilon_z. \tag{13b}$$

Substituting (11)-(12) into (5), we perform the integration over k_z. The poles of the integrand occur at the zeros of $D_1(\bar{k})$ and $D_2(\bar{k})$ denoted by $\pm k_z^o$ and $k_z = k_z^{eu}$ and k_z^{ed}, where

$$k_z^o = [\omega^2\mu\varepsilon - k_x^2 - k_y^2]^{1/2} \tag{14a}$$

$$\begin{Bmatrix} k_z^{eu} \\ k_z^{ed} \end{Bmatrix} = -\frac{\varepsilon_{23}}{\varepsilon_{33}} k_y \pm \frac{1}{\varepsilon_{33}} [\varepsilon_{23}^2 k_y^2 - \varepsilon_{11}\varepsilon_{33}k_x^2 - \varepsilon_{22}\varepsilon_{33}k_y^2 + \omega^2\mu\varepsilon\varepsilon_z\varepsilon_{33}]^{1/2}. \tag{14b}$$

Here, k_z^o (or $-k_z^o$) and k_z^{eu} (or k_z^{ed}) are the z-components of the upward (or downward) propagation vectors for an ordinary wave and an extraordinary wave, respectively. We also notice that the dispersion relations, $D_1(\bar{k}) = 0$ and $D_2(\bar{k}) = 0$, can be obtained by simply transforming the $\bar{k}$-variables in the dispersion relations of vertically uniaxial medium[2], i.e. by replacing k_x, k_y, k_z by k_x, $k_y \cos\psi - k_z \sin\psi$, $k_y \sin\psi + k_z \cos\psi$ in (A11) and (A12) of Ref. [2] which are shown here:

$$D_1^{(0)}(\bar{k}) = k_x{}^2 + k_y{}^2 + k_z{}^2 - \omega^2\mu\varepsilon \tag{15a}$$

$$D_2^{(0)}(\bar{k}) = \varepsilon(k_x{}^2 + k_y{}^2) + \varepsilon_z k_z{}^2 - \omega^2\mu\varepsilon\varepsilon_z . \tag{15b}$$

Assuming the medium to be slightly lossy, i.e. $\mathrm{Im}\, k_z \ll \mathrm{Re}\, k_z$, $\mathrm{Im}\, k_z{}^o > 0$ and $\mathrm{Im}\, k_z^{eu} > 0$, and performing the contour integration over k_z, we obtain for $z > z'$

$$\bar{\bar{G}}(\bar{r}, \bar{r}') = \frac{i}{8\pi^2} \iint_{-\infty}^{\infty} dk_x dk_y \left\{ \frac{e^{i\bar{k}^o \cdot (\bar{r}-\bar{r}')}(\varepsilon - \varepsilon_z)}{k_z{}^o\ D_2(k_z = k_z{}^o)} \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \right.$$

$$\left. + \frac{e^{i\bar{k}^e \cdot (\bar{r}-\bar{r}')}(\varepsilon_z - \varepsilon)/\varepsilon}{(k_z^{eu} - k_z^{ed})\ \omega^2\mu\varepsilon_{33}\ D_1(k_z = k_z^{eu})} \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \right\} \tag{16}$$

where

$$\bar{k}^o = k_x\hat{x} + k_y\hat{y} + k_z{}^o\hat{z} \tag{17a}$$

$$\bar{k}^e = k_x\hat{x} + k_y\hat{y} + k_z^{eu}\hat{z} \tag{17b}$$

$$B_{11} = (k_y \cos\psi - k_z{}^o \sin\psi)^2 \tag{18a}$$

$$B_{12} = B_{21} = -k_x \cos\psi \ \text{[illegible]}\ \cos \ \text{[illegible]}\ k_z{}^o \sin\psi) \tag{18b}$$

$$B_{13} = B_{31} = k_x \sin\psi\ (k_y \cos\psi - k_z{}^o \sin\psi) \tag{18c}$$

$$B_{22} = k_x{}^2 \cos^2\psi \tag{18d}$$

$$B_{23} = B_{32} = -k_x{}^2 \sin\psi \cos\psi \tag{18e}$$

$$B_{33} = k_x{}^2 \sin^2\psi \tag{18f}$$

$$C_{11} = k_x{}^2(k_y \sin\psi + k_z^{eu} \cos\psi)^2 \tag{19a}$$

$$C_{12} = C_{21} = k_x(k_y \sin\psi + k_z^{eu} \cos\psi)[k_y(k_y \sin\psi + k_z^{eu} \cos\psi) - \omega^2\mu\varepsilon \sin\psi] \tag{19b}$$

$$C_{13} = C_{31} = k_x(k_y \sin\psi + k_{1z}^{eu} \cos\psi)[k_z^{eu}(k_y \sin\psi + k_z^{eu} \cos\psi) - \omega^2\mu\varepsilon \cos\psi] \tag{19c}$$

$$C_{22} = [k_y(k_y \sin\psi + k_z^{eu} \cos\psi) - \omega^2\mu\varepsilon \sin\psi]^2 \tag{19d}$$

$$C_{23} = C_{32} = [k_y(k_y \sin\psi + k_z^{eu} \cos\psi) - \omega^2\mu\varepsilon \sin\psi][k_z^{eu}(k_y \sin\psi + k_z^{eu} \cos\psi) - \omega^2\mu\varepsilon \cos\psi] \tag{19e}$$

$$C_{33} = [k_z^{eu}(k_y \sin\psi + k_z^{eu} \cos\psi) - \omega^2\mu\varepsilon \cos\psi]^2. \tag{19f}$$

The following relations are used in the derivation of (16)-(19).

$$\varepsilon[k_x{}^2 + (k_y \cos\psi - k_z{}^e \sin\psi)^2] = -\varepsilon_z[(k_y \sin\psi + k_z{}^e \cos\psi)^2 - \omega^2\mu\varepsilon] \tag{20a}$$

$$\varepsilon[k_x{}^2 + k_y{}^2 + k_z^{e2} - \omega^2\mu\varepsilon_z] = (\varepsilon - \varepsilon_z)(k_y \sin\psi + k_z{}^e \cos\psi)^2 \tag{20b}$$

$$D_2(k_z = k_z^{o}) = (\varepsilon - \varepsilon_z)[k_x^2 + (k_y \cos\psi - k_z^{o} \sin\psi)^2] \tag{21a}$$

$$D_1(k_z = k_z^{e}) = \frac{\varepsilon - \varepsilon_z}{\varepsilon}[(k_y \sin\psi + k_z^{e} \cos\psi)^2 - \omega^2\mu\varepsilon] \tag{21b}$$

$$k_z^{e} = k_z^{eu} \quad \text{or} \quad k_z^{ed}$$

$$\varepsilon_{22} \sin\psi + \varepsilon_{23} \cos\psi = \varepsilon_z \sin\psi \tag{22a}$$

$$\varepsilon_{23} \sin\psi + \varepsilon_{33} \cos\psi = \varepsilon_z \cos\psi \tag{22b}$$

$$\varepsilon_{22} \cos\psi - \varepsilon_{23} \sin\psi = \varepsilon \cos\psi \tag{22c}$$

$$\varepsilon_{23} \cos\psi - \varepsilon_{33} \cos\psi = -\varepsilon \sin\psi \tag{22d}$$

$$\varepsilon - \varepsilon_{22} = (\varepsilon - \varepsilon_z) \sin^2\psi \tag{23a}$$

$$\varepsilon_z - \varepsilon_{22} = (\varepsilon_z - \varepsilon) \cos^2\psi \tag{23b}$$

$$\varepsilon - \varepsilon_{33} = (\varepsilon - \varepsilon_z) \cos^2\psi \tag{23c}$$

$$\varepsilon_z - \varepsilon_{33} = (\varepsilon_z - \varepsilon) \sin^2\psi \tag{23d}$$

For $z < z'$, we obtain $\bar{\bar{G}}(\bar{r}, \bar{r}')$ in a similar manner by noticing that $\mathrm{Im}(-k_z^{o}) < 0$ and $\mathrm{Im}(k_z^{ed}) < 0$.

Finally we cast the dyadic Green's function for the unbounded anisotropic medium, $\bar{\bar{G}}(\bar{r}, \bar{r}')$ in the following form:

$$\bar{\bar{G}}(\bar{r}, \bar{r}') = \frac{i}{8\pi^2} \iint_{-\infty}^{\infty} dk_x dk_y \left\{ \frac{1}{k_z{}^o} \hat{o}(k_z{}^o)\, \hat{o}(k_z{}^o)\, e^{i\bar{K}^o \cdot (\bar{r}-\bar{r}')} \right.$$

$$\left. + \frac{2}{k_z^{eu} - k_z^{ed}} \frac{\omega^2\mu(\varepsilon + \varepsilon_z) - (k_x{}^2 + k_y{}^2 + k_z^{eu\,2})}{\omega^2\mu\varepsilon_{33}} \hat{e}(k_z^{eu})\, \hat{e}(k_z^{eu})\, e^{i\bar{K}^e \cdot (\bar{r}-\bar{r}')} \right\}$$

$$\text{for} \quad z > z' \tag{24a}$$

$$\bar{\bar{G}}(\bar{r}, \bar{r}') = \frac{i}{8\pi^2} \iint_{-\infty}^{\infty} dk_x dk_y \left\{ \frac{1}{k_z{}^o} \hat{o}(-k_z{}^o)\, \hat{o}(-k_z{}^o)\, e^{i\bar{K}^o \cdot (\bar{r}-\bar{r}')} \right.$$

$$\left. + \frac{2}{k_z^{eu} - k_z^{ed}} \frac{\omega^2\mu(\varepsilon + \varepsilon_z) - (k_x{}^2 + k_y{}^2 + k_z^{ed\,2})}{\omega^2\mu\varepsilon_{33}} \hat{e}(k_z^{ed})\, \hat{e}(k_z^{ed})\, e^{i\bar{K}^e \cdot (\bar{r}-\bar{r}')} \right\}$$

$$\text{for} \quad z < z' \tag{24b}$$

where

$$\hat{o}(k_z{}^o) = \frac{\hat{z}' \times \bar{k}^o}{|\hat{z}' \times \bar{k}^o|} \tag{25a}$$

$$\hat{o}(-k_z{}^o) = \frac{\hat{z}' \times \bar{K}_1{}^o}{|\hat{z}' \times \bar{K}_1{}^o|} \tag{25b}$$

$$\hat{e}(k_z^{eu}) = \frac{\hat{o}(k_z^{eu}) \times \bar{K}_u}{|\bar{K}_u|} \tag{25c}$$

$$\hat{e}(k_z^{ed}) = \frac{\hat{o}(k_z^{ed}) \times \bar{K}_u}{|\bar{K}_u|} \tag{25d}$$

$$\hat{o}(k_z^{eu}) = \frac{\hat{z}' \times \bar{k}^e}{|\hat{z}' \times \bar{k}^e|} \tag{25e}$$

$$\hat{o}(k_z^{ed}) = \frac{\hat{z}' \times \bar{k}^e}{|\hat{z}' \times \bar{k}^e|} \tag{25f}$$

$$\bar{k}^o = \bar{k}_\rho - k_z^o\hat{z}$$

$$\bar{k}^e = \bar{k}_\rho + k_z^{ed}\hat{z}$$

$$\bar{k}_u = \bar{\bar{\varepsilon}} \cdot \bar{k}^e$$

$$\bar{K}_u = \bar{\bar{\varepsilon}} \cdot \bar{K}^e$$

$$\bar{k}_\rho = k_x\hat{x} + k_y\hat{y}$$

$$\hat{z}' = \hat{y}\sin\psi + \hat{z}\cos\psi.$$

We notice that $\hat{o}(\pm k_z^o)$ is a unit vector in the direction of the electric field for an ordinary wave and $\hat{e}(k_z^{eu})$ or $\hat{e}(k_z^{ed})$ is a unit vector in the direction of the electric field for an extraordinary wave. It is seen that $\hat{o}$ is linearly polarized perpendicular to the plane formed by the optic axis (z'-axis) and the propagation vector ($\bar{k}^o$-vector) as it represents an o-wave, and $\hat{e}$ is linearly polarized parallel to the plane formed by z'-axis and $\bar{k}^e$-vector as it is an e-wave[2]. We also notice that for an o-wave, the direction of power flow determined by the Poynting vector $\bar{E} \times \bar{H}^*$ is the same as the direction of propagation vector ($\bar{k}^o$), while for an e-wave the direction of power flow is different from that of propagation vector ($\bar{k}^e$). In fact, for an upward propagating e-wave, the electric field ($\bar{E}$), the magnetic field ($\bar{H}$) and the <u>Poynting vector ($\bar{E} \times \bar{H}^*$)</u> are in the direction of

$\hat{e}(k_z^e)$, $\hat{o}(k_z^e)$ and $\bar{k}_u$, respectively.

The result checks with the expression of DGF for a vertically uniaxial medium[1] in the limiting case as $\psi = 0$. For an isotropic medium where $\psi = 0$ and $\epsilon = \epsilon_z$, the o-wave becomes a horizontally polarized TE wave and the e-wave becomes a vertically polarized TM wave as follows.

$$\hat{o}(\pm k_z^o) \rightarrow \hat{h}(\pm k_z) = \frac{1}{k_\rho} \hat{z} \times \bar{k} \tag{26a}$$

$$\hat{e}(k_z^{eu}) \rightarrow \hat{v}(k_z) = \frac{1}{k} \hat{h}(k_z) \times \bar{k} \tag{26b}$$

$$\hat{e}(k_z^{ed}) \rightarrow \hat{v}(-k_z) = \frac{1}{k} \hat{h}(-k_z) \times \bar{K} \tag{26c}$$

where

$$k_\rho = (k_x^2 + k_y^2)^{1/2} \tag{27a}$$

$$k_z = (k^2 - k_\rho^2)^{1/2} \tag{27b}$$

$$k = \omega\sqrt{\mu\epsilon} \tag{27c}$$

$$\bar{k} = \bar{k}_\rho + k_z\hat{z} \tag{27d}$$

$$\bar{K} = \bar{k}_\rho - k_z\hat{z}. \tag{27e}$$

III. DGF for Two-Layered Anisotropic Medium

Next we consider the two-layered stratified medium as shown in Fig. 2. The upper and lower media are isotropic and characterized by ε_0 and ε_2, respectively. The medium in the middle is anisotropic as described in the previous section. In the principal coordinate system it is characterized by ε_1 and ε_{1z}. The permittivity tensor after the tilting of the optic axis is given by

$$\bar{\bar{\varepsilon}}_1 = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & \varepsilon_{23} \\ 0 & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} . \tag{28}$$

where

$$\varepsilon_{11} = \varepsilon_1, \tag{29a}$$

$$\varepsilon_{22} = \varepsilon_1 \cos^2\psi + \varepsilon_{1z} \sin^2\psi \tag{29b}$$

$$\varepsilon_{23} = \varepsilon_{32} = (\varepsilon_{1z} - \varepsilon_1) \sin\psi \cos\psi \tag{29c}$$

$$\varepsilon_{33} = \varepsilon_1 \sin^2\psi + \varepsilon_{1z} \cos^2\psi. \tag{29d}$$

All three regions are assumed to have the same permeability μ. Let the source be in region 0. DGF's $\bar{\bar{G}}_{00}(\bar{r}, \bar{r}')$, $\bar{\bar{G}}_{10}(\bar{r}, \bar{r}')$ and $\bar{\bar{G}}_{20}(\bar{r}, \bar{r}')$ satisfy

$$\nabla \times \nabla \times \bar{\bar{G}}_{00}(\bar{r}, \bar{r}') - \omega^2\mu\epsilon_0 \bar{\bar{G}}_{00}(\bar{r}, \bar{r}') = \bar{\bar{I}}\, \delta(\bar{r} - \bar{r}'), \qquad z \geq 0 \qquad (30a)$$

$$\nabla \times \nabla \times \bar{\bar{G}}_{10}(\bar{r}, \bar{r}') - \omega^2\mu\bar{\bar{\epsilon}}_1 \cdot \bar{\bar{G}}_{10}(\bar{r}, \bar{r}') = 0, \qquad -d \leq z \leq 0 \qquad (30b)$$

$$\nabla \times \nabla \times \bar{\bar{G}}_{20}(\bar{r}, \bar{r}') - \omega^2\mu\epsilon_2 \bar{\bar{G}}_{20}(\bar{r}, \bar{r}') = 0, \qquad z \leq -d \qquad (30c)$$

and the appropriate boundary conditions at $z = 0$ and $z = -d$ and radiation conditions at $z = \pm\infty$. The first subscript of the DGF refers to the region of the field and the second subscript to the region containing the source.

Since the DGF for an infinite unbounded media assumes the form in (24), it is appropriate to assume the DGF's for this two-layered geometry in the following form:

$$\bar{\bar{G}}_{00}(\bar{r}, \bar{r}') = \frac{i}{8\pi^2} \iint_{-\infty}^{\infty} dk_x dk_y \frac{1}{k_{oz}} \Big\{ [\hat{h}_o(-k_{oz})\, e^{i\bar{K}_o \cdot \bar{r}} + R_{HH}\, \hat{h}_o(k_{oz})\, e^{i\bar{K}_o \cdot \bar{r}}$$

$$+ R_{HV}\, \hat{v}_o(k_{oz})\, e^{i\bar{K}_o \cdot \bar{r}}]\, \hat{h}_o(-k_{oz}) + [\hat{v}_o(-k_{oz})\, e^{i\bar{K}_o \cdot \bar{r}}$$

$$+ R_{VV}\, \hat{v}_o(k_{oz})\, e^{i\bar{K}_o \cdot \bar{r}} + R_{VH}\, \hat{h}(k_{oz})\, e^{i\bar{K}_o \cdot \bar{r}}]\, \hat{v}_o(-k_{oz}) \Big\}\, e^{-i\bar{K}_o \cdot \bar{r}'}$$

$$0 \leq z \leq z' \qquad (31a)$$

$$\bar{\bar{G}}_{10}(\bar{r}, \bar{r}') = \frac{i}{8\pi^2} \iint_{-\infty}^{\infty} dk_x dk_y \frac{1}{k_{oz}} \Big\{ [A_{HO}\, \hat{o}(-k_{1z}^{o})\, e^{i\bar{K}_1^{o} \cdot \bar{r}} + B_{HO}\, \hat{o}(k_{1z}^{o})\, e^{i\bar{K}_1^{o} \cdot \bar{r}}$$

$$+ A_{He}\,\hat{e}(k_{1z}^{ed})\,e^{i\bar{K}_1^e\cdot\bar{r}} + B_{He}\,\hat{e}(k_{1z}^{eu})\,e^{i\bar{K}_1^e\cdot\bar{r}}\,]\,\hat{h}_o(-k_{oz})$$

$$+ [A_{Vo}\,\hat{o}(-k_{1z}^{o})\,e^{i\bar{K}_1^o\cdot\bar{r}} + B_{Vo}\,\hat{o}(k_{1z}^{o})\,e^{i\bar{K}_1^o\cdot\bar{r}}$$

$$+ A_{Ve}\,\hat{e}(k_{1z}^{ed})\,e^{i\bar{K}_1^e\cdot\bar{r}} + B_{Ve}\,\hat{e}(k_{1z}^{eu})\,e^{i\bar{K}_1^e\cdot\bar{r}}\,]\,\hat{v}_o(-k_{oz})\Big\}\,e^{-i\bar{K}_o\cdot\bar{r}'},$$

$$-d \le z \le 0 \qquad (31b)$$

$$\bar{\bar{G}}_{20}(\bar{r},\bar{r}') = \frac{i}{8\pi^2}\iint_{-\infty}^{\infty} dk_x dk_y\,\frac{1}{k_{oz}}\Big\{[X_{HH}\,\hat{h}_2(-k_{2z}) + X_{HV}\,\hat{v}_2(-k_{2z})]\,\hat{h}_o(-k_{oz})$$

$$+ [X_{VV}\,\hat{v}_2(-k_{2z}) + X_{VH}\,\hat{h}_2(-k_{2z})]\,\hat{v}_o(-k_{oz})\Big\}\,e^{i\bar{K}_1\cdot\bar{r}}\,e^{-i\bar{K}_o\cdot\bar{r}'},$$

$$z \le -d \qquad (31c)$$

where

$$\hat{h}_i(\pm k_{iz}) = \frac{1}{k_\rho}\,\hat{z}\times k_i, \qquad i = 0, 2 \qquad (32a)$$

$$\hat{v}_i(k_{iz}) = \frac{1}{k_i}\,\hat{h}_i(k_{iz})\times\bar{K}_i, \qquad i = 0, 2 \qquad (32b)$$

$$\hat{v}_i(-k_{iz}) = \frac{1}{k_i}\,\hat{h}_i(-k_{iz})\times\bar{K}_i, \qquad i = 0, 2 \qquad (32c)$$

$$\hat{o}(k_{1z}^{o}) = \frac{\hat{z}'\times\bar{K}_1^{o}}{|\hat{z}'\times\bar{K}_1^{o}|}$$

$$\hat{o}(-k_{1z}^{o}) = \frac{\hat{z}'\times\bar{K}_1^{o}}{|\hat{z}'\times\bar{K}_1^{o}|}$$

$$\hat{e}(k_{1z}^{eu}) = \frac{\hat{o}(k_{1z}^{eu}) \times \bar{k}_{1u}}{|\bar{k}_{1u}|}$$

$$\hat{e}(k_{1z}^{ed}) = \frac{\hat{o}(k_{1z}^{ed}) \times \bar{K}_{1u}}{|\bar{K}_{1u}|}$$

$$\bar{k}_i = \bar{k}_\rho + k_{iz}\hat{z}, \qquad i = 0,\ 2$$

$$\bar{K}_i = \bar{k}_\rho - k_{iz}\hat{z}, \qquad i = 0,\ 2$$

$$\bar{k}_1{}^{o} = \bar{k}_\rho + k_{1z}^{o}\hat{z}$$

$$\bar{K}_1{}^{o} = \bar{k}_\rho - k_{1z}^{o}\hat{z}$$

$$\bar{k}_1{}^{e} = \bar{k}_\rho + k_{1z}^{eu}\hat{z}$$

$$\bar{K}_1{}^{e} = \bar{k}_\rho + k_{1z}^{ed}\hat{z}$$

$$\bar{k}_{1u} = \bar{\bar{\varepsilon}}_1 \cdot \bar{k}_1{}^{e}$$

$$\bar{K}_{1u} = \bar{\bar{\varepsilon}}_1 \cdot \bar{K}_1{}^{e}$$

$$k_{iz} = \sqrt{k_i{}^2 - k_\rho{}^2}, \qquad i = 0,\ 2$$

$$k_{1z}^{o} = \sqrt{k_1{}^2 - k_\rho{}^2}$$

$$\left\{\begin{matrix} k_{1z}^{eu} \\ k_{1z}^{ed} \end{matrix}\right\} = -\frac{\varepsilon_{23}}{\varepsilon_{33}} k_y \pm \frac{1}{\varepsilon_{33}} [\varepsilon_{23}^2 k_y{}^2 - \varepsilon_{11}\varepsilon_{33}k_x{}^2 - \varepsilon_{22}\varepsilon_{33}k_y{}^2 + k_1{}^2\varepsilon_{1z}\varepsilon_{33}]^{1/2}$$

$$k_i = \omega\sqrt{\mu\varepsilon_i}, \qquad i = 0, 1, 2.$$

The boundary conditions to be satisfied at $z = 0$ and $z = -d$ are

$$\hat{z} \times \bar{\bar{G}}_{00}(\bar{r}, \bar{r}') = \hat{z} \times \bar{\bar{G}}_{10}(\bar{r}, \bar{r}'), \qquad z = 0 \tag{33a}$$

$$\hat{z} \times \bar{\bar{G}}_{10}(\bar{r}, \bar{r}') = \hat{z} \times \bar{\bar{G}}_{20}(\bar{r}, \bar{r}'), \qquad z = -d \tag{33b}$$

$$\hat{z} \times \nabla \times \bar{\bar{G}}_{00}(\bar{r}, \bar{r}') = \hat{z} \times \nabla \times \bar{\bar{G}}_{10}(\bar{r}, \bar{r}') \qquad z = 0 \tag{33c}$$

$$\hat{z} \times \nabla \times \bar{\bar{G}}_{10}(\bar{r}, \bar{r}') = \hat{z} \times \nabla \times \bar{\bar{G}}_{20}(\bar{r}, \bar{r}') \qquad z = -d \tag{33d}$$

where (33a) and (33b) refer to the continuity of tangential electric fields and (33c) and (33d) refer to the continuity of tangential magnetic fields at each interface. Substituting (31) into (33), we obtain eight linear algebraic equations for the eight unknown coefficients. However, it is very difficult to solve these equations analytically and to express the unknowns in a simple form. Instead, we will express these two-layer coefficients in terms of the half-space reflection and transmission coefficients using a matrix method.

Let the amplitude vectors of the incident and reflected waves in region 0 be a and b, respectively, as shown in Fig. 3. A and B are the amplitude vectors of the upward and downward propagating waves in region 1, respectively. C is the amplitude vector of the transmitted wave in region 2. Then

these amplitude vectors satisfy the following matrix equations

$$\begin{pmatrix} b \\ A \end{pmatrix} = \begin{pmatrix} R_{01} & T_{10} \\ T_{01} & R_{10} \end{pmatrix} \begin{pmatrix} a \\ B \end{pmatrix} \tag{34a}$$

$$\begin{pmatrix} B \\ C \end{pmatrix} = \begin{pmatrix} R_{12} \\ T_{12} \end{pmatrix} A \tag{34b}$$

where R_{01}, T_{01}, R_{10}, T_{10}, R_{12}, T_{12} are the half-space reflection and transmission matrices. It should be noticed that R_{12} and T_{12} also account for the phase difference because the boundary is shifted. Essentially (34a) and (34b) reflect the boundary conditions at $z = 0$ and $z = -d$, respectively. From (34b),

$$B = R_{12}A \tag{35a}$$

$$C = T_{12}A. \tag{35b}$$

From (33a),

$$b = R_{01}a + T_{10}B \tag{36a}$$

$$A = T_{01}a + R_{10}B \tag{36b}$$

Substituting (35a) into (36b), we express A in terms of a:

$$A = (I - R_{10}R_{12})^{-1} T_{01}a. \tag{37}$$

Combining (35a), (36a), and (37), b is given by

$$b = \{R_{10} + T_{10}R_{12}(I - R_{10}R_{12})^{-1} T_{01}\} a. \tag{38}$$

If we let

$$b \equiv Ra \tag{39a}$$

$$A \equiv Da \tag{39b}$$

$$B \equiv Ua \tag{39c}$$

$$C \equiv Ta \tag{39d}$$

then R, D, U and T are considered as the reflection matrix, downward propagation matrix, upward propagation matrix, and transmission matrix, respectively. Using the equations (35)-(38), we obtain

$$R = R_{01} + T_{10}R_{12}(I - R_{10}R_{12})^{-1} T_{01} \tag{40a}$$

$$D = (I - R_{10}R_{12})^{-1} T_{01} \tag{40b}$$

$$U = R_{12}(I - R_{10}R_{12})^{-1} T_{01} \tag{40c}$$

$$T = T_{12}(I - R_{10}R_{12})^{-1} T_{01}. \tag{40d}$$

The components of the two-layer matrices are written as

$$R = \begin{bmatrix} R_{HH} & R_{VH} \\ R_{HV} & R_{VV} \end{bmatrix} \tag{41a}$$

$$D = \begin{bmatrix} A_{Ho} & A_{Vo} \\ A_{He} & A_{Ve} \end{bmatrix} \tag{41b}$$

$$U = \begin{bmatrix} B_{Ho} & B_{Vo} \\ B_{He} & B_{Ve} \end{bmatrix} \tag{41c}$$

$$T = \begin{bmatrix} X_{HH} & X_{VH} \\ X_{HV} & X_{VV} \end{bmatrix} . \tag{41d}$$

The half-space reflection and transmission matrices are expressed as

$$R_{01} = \begin{bmatrix} R_{01HH} & R_{01VH} \\ R_{01HV} & R_{01VV} \end{bmatrix} \tag{42a}$$

$$T_{01} = \begin{bmatrix} X_{Ho} & X_{Vo} \\ X_{He} & X_{Ve} \end{bmatrix} \tag{42b}$$

$$R_{10} = \begin{bmatrix} R_{oo} & R_{eo} \\ R_{oe} & R_{ee} \end{bmatrix} \tag{42c}$$

$$T_{10} = \begin{bmatrix} X_{oH} & X_{eH} \\ X_{oV} & X_{eV} \end{bmatrix} \tag{42d}$$

$$R_{12} = \begin{bmatrix} e^{ik_{1z}^{o}d} R_{12oo}\, e^{ik_{1z}^{o}d} & e^{-ik_{1z}^{ed}d} R_{12eo}\, e^{ik_{1z}^{o}d} \\ e^{ik_{1z}^{o}d} R_{12oe}\, e^{ik_{1z}^{eu}d} & e^{-ik_{1z}^{ed}d} R_{12ee}\, e^{ik_{1z}^{eu}d} \end{bmatrix} \equiv \begin{bmatrix} o_o & e_o \\ o_e & e_o \end{bmatrix} \tag{42e}$$

$$T_{12} = \begin{bmatrix} e^{ik_{1z}^{o}d} X_{12oH}\, e^{-ik_{2z}d} & e^{-ik_{1z}^{ed}d} X_{12eH}\, e^{-ik_{2z}d} \\ e^{ik_{1z}^{o}d} X_{12oV}\, e^{-ik_{2z}d} & e^{-ik_{1z}^{ed}d} X_{12eV}\, e^{-ik_{2z}d} \end{bmatrix} \equiv \begin{bmatrix} t_{oH} & t_{eH} \\ t_{oV} & t_{eV} \end{bmatrix} . \tag{42f}$$

The equations which all these half-space coefficients govern and the expressions for them are shown in Appendix A. Substituting (42) into (40), we finally obtain the expressions of two-layer coefficients in terms of half-space coefficients.

$$\begin{aligned} R_{\beta\alpha} = {} & R_{01\beta\alpha} + X_{\beta o}(L_1 o_o + M_2 e_o)X_{o\alpha} + X_{\beta o}(L_1 o_e + M_2 e_e)X_{e\alpha} \\ & + X_{\beta e}(L_2 o_o + M_1 e_o)X_{o\alpha} + X_{\beta e}(L_2 o_e + M_1 e_e)X_{e\alpha} \end{aligned} \tag{43a}$$

$$A_{\beta o} = X_{\beta o}L_1 + X_{\beta e}L_2 \tag{43b}$$

$$A_{\beta e} = X_{\beta o}M_2 + X_{\beta e}M_1 \tag{43c}$$

$$B_{\beta\gamma} = X_{\beta o}(L_1 o_\gamma + M_2 e_\gamma) + X_{\beta e}(L_2 o_\gamma + M_1 e_\gamma) \tag{43d}$$

$$X_{\beta\alpha} = X_{\beta o}(L_1 t_{o\alpha} + M_2 t_{e\alpha}) + X_{\beta e}(L_2 t_{o\alpha} + M_1 t_{e\alpha}) \tag{43e}$$

where

$$L_1 = \frac{1 - S_{ee}}{D} \tag{44a}$$

$$L_2 = \frac{S_{eo}}{D} \tag{44b}$$

$$M_1 = \frac{1 - S_{oo}}{D} \tag{44c}$$

$$M_2 = \frac{S_{oe}}{D} \tag{44d}$$

$$S_{oo} = o_o R_{oo} + o_e R_{eo} \tag{44e}$$

$$S_{oe} = o_o R_{oe} + o_e R_{ee} \tag{44f}$$

$$S_{eo} = e_o R_{oo} + e_e R_{eo} \tag{44g}$$

$$S_{ee} = e_o R_{oe} + e_e R_{ee} \tag{44h}$$

$$D = (1 - S_{oo})(1 - S_{ee}) - S_{oe}S_{eo} \tag{44i}$$

$\beta, \alpha = H$ or V

$r = o$ or e.

The expressions in (43) can also be derived independently by using the method of series expansion where we express each two-layer coefficient as an infinite series in terms of half-space coefficients by identifying all the constituents and summing the series.

IV. Reflection and Transmission Coefficients

First, consider the wave with TE polarization incident from a free space (region 0) upon an anisotropic medium (region 1) as shown in Fig. 4a. The electric fields in each region can be formulated as follows

$$E_0(\bar{r}) = \hat{h}_0(-k_{oz})\, e^{i\bar{K}_o \cdot \bar{r}} + R_{01HH}\, \hat{h}_0(k_{oz})\, e^{i\bar{K}_o \cdot \bar{r}} + R_{01HV}\, \hat{v}_0(k_{oz})\, e^{i\bar{K}_o \cdot \bar{r}}, \qquad z \geq 0 \tag{45a}$$

$$E_1(\bar{r}) = X_{Ho}\, \hat{o}(-k_{1z}^{o})\, e^{i\bar{K}_1^{o} \cdot \bar{r}} + X_{He}\, \hat{e}(k_{1z}^{ed})\, e^{i\bar{K}_1^{e} \cdot r}, \qquad z \leq 0. \tag{45b}$$

The boundary conditions to be satisfied at $z = 0$ are

$$\hat{z} \times E_0(\bar{r}) = \hat{z} \times E_1(\bar{r}) \tag{46a}$$

$$\hat{z} \times \nabla \times E_0(\bar{r}) = \hat{z} \times \nabla \times E_1(\bar{r}). \tag{46b}$$

Substituting (45) into (46), we obtain four algebraic equations. Solving them for four unknowns, we obtain

$$X_{He} = -\frac{U_d}{D_e}\frac{2k_{oz}}{k_\rho}\, k_x(k_\rho{}^2 + k_{oz}k_{1z}^{o})\sin\psi$$

where

$$D_e = \cos^2\psi\, k_\rho{}^2(k_1{}^2 k_{oz} - k_o{}^2 k_{1z}^{ed}) + \sin^2\psi\{k_1{}^2(k_{oz} - k_{1z}^{ed})(k_x{}^2 + k_{oz}k_{1z}^{o})$$

$$+ k_y{}^2 k_{1z}(k_1{}^2 - k_o{}^2)\} + \cos\psi \sin\psi\, k_y(k_{1z}^{o} + k_{oz})(k_e{}^{o} - k_{1z}^{ed})$$

$$(k_\rho{}^2 + k_{1z}^{o}k_{oz}),$$

$$U_d = \left\{\frac{\varepsilon_1}{\varepsilon_1 - \varepsilon_{1z}}(k_x{}^2 + k_y{}^2 + k_z^{ed_2} - \omega^2\mu\varepsilon_1)[k_x{}^2 + k_y{}^2 + k_{1z}^{ed_2}\right.$$

$$\left. - \omega^2\mu(\varepsilon_1 + \varepsilon_{1z})]\right\}^{1/2}.$$

$$X_{HO} = \frac{g_d}{D_e}\frac{1}{(k_{oz} + k_{1z}^{o})} 2\frac{k_{oz}}{k_\rho}\{k_\rho{}^2(k_1{}^2 k_{oz} - k_o{}^2 k_{1z}^{ed}) \cos\psi$$

$$+ k_y(k_o{}^2 k_{1z}^{o2} - k_1{}^2 k_z k_{1z}^{ed}) \sin\psi\}$$

where

$$g_d = \{k_x{}^2 + (k_y \cos\psi + k_{1z}^{o} \sin\psi)^2\}^{1/2}$$

$$R_{01HH} = -1 + \frac{X_o}{g_d}\frac{1}{k_\rho}(k_\rho{}^2 \cos\psi + k_y k_{1z}^{o} \sin\psi) - \frac{k_1{}^2}{k_\rho} k_x \sin\psi \frac{X_e}{U_d}$$

$$R_{01VV} = -\frac{X_o}{g_d}\frac{k_o k_x}{k_\rho k_{oz}} k_{1z}^{o} \sin\psi + \frac{X_e}{U_d}\frac{k_o}{k_\rho k_{oz}}(k_\rho{}^2 k_{1z}^{ed} \cos\psi - k_{1z}^{o2} k_y \sin\psi).$$

Second, for the incident wave with TM polarization [Fig. 4b], the electric fields satisfy

$$\bar{E}_o(\bar{r}) = \hat{v}_o(-k_{oz})\, e^{i\bar{K}_o\cdot\bar{r}} + R_{01VV}\, \hat{v}_o(k_{oz})\, e^{i\bar{K}_o\cdot\bar{r}} + R_{01VH}\, \hat{h}_o(k_{oz})\, e^{i\bar{K}_o\cdot\bar{r}}, \qquad z \geq 0$$

$$\bar{E}_1(\bar{r}) = X_{Vo}\, \hat{o}(-k^o_{1z})\, e^{i\bar{K}_1{}^o\cdot\bar{r}} + X_{He}\, \hat{e}(k^{ed}_{1z})\, e^{i\bar{K}_1{}^e\cdot\bar{r}}, \qquad z \leq 0.$$

Applying the same boundary conditions as (46), we obtain

$$X_{Ve} = \frac{U_d}{D_e}\frac{2k_o}{k_\rho}(k_{oz}k_\rho{}^2 \cos\psi + k_y k_{oz} k^o_{1z} \sin\psi)$$

$$X_{Vo} = \frac{g_d}{D_e}\frac{2k_o k_{oz} k_1{}^2 k_x}{k_\rho}\frac{k_{oz} - k^{ed}_{1z}}{k_{oz} + k^o_{1z}} \sin\psi$$

$$R_{01VH} = \frac{Y_o}{g_d}\frac{1}{k_\rho}(k_\rho{}^2 \cos\psi + k_y k^o_{1z} \sin\psi) - \frac{k_1{}^2}{k_\rho} k_x \sin\psi \frac{Y_e}{U_d}$$

$$R_{01VV} = 1 - \frac{Y_o}{g_d}\frac{k_o k_x}{k_\rho k_{oz}} k^o_{1z} \sin\psi + \frac{Y_e}{U_d}\frac{k_o}{k_\rho k_{oz}}(k_\rho{}^2 k^{ed}_{1z} \cos\psi - k^{o2}_{1z} k_y \sin\psi).$$

Now consider the wave incident from an anisotropic medium (region 1) upon an isotropic medium (region 2). For the ordinary wave incident as shown in Fig. 5a, the electric fields satisfy

$$\bar{E}_1(\bar{r}) = \hat{o}(-k^o_{1z})\, e^{i\bar{K}_1\cdot\bar{r}} + R_{12oo}\, \hat{o}(k^o_{1z})\, e^{i\bar{K}_1{}^o\cdot\bar{r}} + R_{12oe}\, \hat{e}(k^{eu}_{1z})\, e^{i\bar{K}_1{}^e\cdot\bar{r}}, \qquad z \geq 0$$

$$\bar{E}_2(\bar{r}) = X_{12oH}\,\hat{h}_2(-k_{2z})\,e^{i\bar{K}_2\cdot\bar{r}} + X_{12oV}\,\hat{v}_2(-k_{2z})\,e^{i\bar{K}_2\cdot\bar{r}}, \qquad z \leq 0.$$

Applying the same boundary conditions, we obtain

$$R_{12oo} = \frac{g_u}{g_d}\frac{G_e}{F_e}\frac{k_{1z}^{o} - k_{2z}}{k_{1z}^{o} + k_{2z}}$$

where

$$G_e = \cos^2\psi\; k_\rho{}^2(k_1{}^2k_{2z} + k_2{}^2k_{1z}^{eu}) + \sin^2\psi\{k_1{}^2(k_{2z} + k_{1z}^{eu})(k_x{}^2 - k_{2z}k_{1z}^{o})$$
$$- k_y{}^2k_{1z}^{o}(k_1{}^2 - k_2{}^2)\} + \cos\psi\,\sin\psi\; k_y(k_{2z} - k_{1z}^{o})(k_{1z}^{o} - k_{1z}^{eu})(k_\rho{}^2 - k_{2z}k_{1z}^{o})$$

$$g_u = \{k_x{}^2 + (k_y\cos\psi - k_{1z}^{o}\sin\psi)^2\}^{1/2}.$$

$$R_{12oe} = \frac{U_u}{g_d}\frac{1}{F_e}\,2k_xk_{1z}^{o}(k_{2z} - k_{1z}^{o})(k_y\sin\psi + k_{2z}\cos\psi)\sin\psi$$

where

$$F_e = \cos^2\psi\; k_\rho{}^2(k_1{}^2k_{2z} + k_2{}^2k_{1z}^{eu}) + \sin^2\psi\{k_1{}^2(k_{2z} + k_{1z}^{eu})(k_x{}^2 + k_{2z}k_{1z}^{o})$$
$$+ k_y{}^2k_{1z}^{o}(k_1{}^2 - k_2{}^2)\} - \cos\psi\,\sin\psi\; k_y(k_{1z}^{o} + k_{2z})(k_{1z}^{o2} + k_{1z}^{eu})(k_\rho{}^2 + k_{1z}^{o}k_{2z})$$

$$U_u = \left\{\frac{\varepsilon_1}{\varepsilon_1 - \varepsilon_{1z}}(k_x{}^2 + k_y{}^2 + k_{1z}^{eu\,2} - \omega^2\mu\varepsilon_1)[k_x{}^2 + k_y{}^2 + k_{1z}^{eu\,2} - \omega^2\mu(\varepsilon_1 + \varepsilon_{1z})]\right\}^{1/2}$$

$$X_{12oH} = \frac{1}{g_d k_\rho}(k_\rho{}^2 \cos\psi + k_y k_{1z}^o \sin\psi) + \frac{R_{12oo}}{g_u k_\rho}(k_\rho{}^2 \cos\psi - k_y k_{1z}^o \sin\psi)$$

$$- \frac{R_{12oe}}{U_u k_\rho} k_1{}^2 k_x \sin\psi$$

$$X_{12oV} = \frac{k_2}{k_{2z} k_\rho}\left\{\frac{1}{g_d} k_x k_{1z}^o \sin\psi - \frac{R_{12oo}}{g_u} k_x k_{1z}^o \sin\psi + \frac{R_{12oe}}{U_u}(k_y k_{1z}^{o2} \sin\psi\right.$$

$$\left. - k_\rho{}^2 k_{1z}^{eu} \cos\psi)\right\} .$$

For the extraordinary incident [Fig. 5b], the $\bar{E}$ fields satisfy

$$\bar{E}_1(\bar{r}) = \hat{e}(k_{1z}^{ed})\, e^{i\bar{K}_1{}^e \cdot \bar{r}} + R_{12ee}\, \hat{e}(k_{1z}^{eu})\, e^{i\bar{k}_1{}^e \cdot \bar{r}} + R_{12eo}\, \hat{o}(k_{1z}^o)\, e^{i\bar{k}_1{}^o \cdot \bar{r}}, \qquad z \geq 0$$

$$\bar{E}_2(\bar{r}) = X_{12eH}\, \hat{h}_2(-k_{2z})\, e^{i\bar{K}_2 \cdot \bar{r}} + X_{12eV}\, \hat{v}_2(-k_{2z})\, e^{i\bar{K}_2 \cdot \bar{r}}, \qquad z \leq 0.$$

Applying the boundary conditions, we get

$$R_{12ee} = -\frac{U_u}{U_d}\frac{H_e}{F_e}$$

where

$$H_e = \cos^2\psi\, k_\rho{}^2(k_1{}^2 k_{2z} + k_2{}^2 k_{1z}^{ed}) + \sin^2\psi\{k_1{}^2(k_{2z} + k_{1z}^{ed})(k_x{}^2 + k_{2z}k_{1z}^o)$$

$$+ k_y{}^2 k_{1z}^{o}(k_1{}^2 - k_2{}^2)\} - \cos\psi \sin\psi\, k_y(k_{1z}^{o} + k_{2z})(k_{1z}^{o} + k_{1z}^{ed})(k_\rho{}^2 + k_{1z}^{o} k_{2z})$$

$$R_{12eo} = \frac{g_u}{U_d}\frac{1}{F_e} k_1{}^2 k_x (k_{1z}^{eu} - k_{1z}^{ed})(k_{1z}^{o} - k_{2z})(k_y \sin\psi - k_{2z}\cos\psi)\sin\psi$$

$$X_{12ev} = \frac{k_2}{k_{2z}k_\rho}\left\{\frac{1}{U_d}(k_{1z}^{o2} k_y \sin\psi - k_\rho{}^2 k_{1z}^{ed}\cos\psi) - \frac{R_{12eo}}{g_u} k_x k_{1z}^{o}\sin\psi \right.$$
$$\left. + \frac{R_{ee}}{U_u}(k_y k_{1z}^{o2}\sin\psi - k_\rho{}^2 k_{1z}^{eu}\cos\psi)\right\}$$

$$X_{12eH} = \frac{1}{k_\rho}\left\{-\frac{k_1{}^2}{U_d} k_x \sin\psi + \frac{R_{12eo}}{g_u}(k_\rho{}^2\cos\psi - k_y k_{1z}^{o}\sin\psi) - \frac{R_{12ee}}{U_u} k_1{}^2 k_x \sin\psi\right\}.$$

As a final configuration, the wave is incident from an anisotropic medium in the lower region 1 upon an isotropic medium in the upper region 0. For the incident ordinary wave [Fig. 6a], the $\bar{E}$ fields satisfy

$$\bar{E}_1(\bar{r}) = \hat{o}(k_{1z}^{o})\, e^{i\bar{K}_1{}^{o}\cdot\bar{r}} + R_{oo}\,\hat{o}(-k_{1z}^{o})\, e^{i\bar{K}_1{}^{o}\cdot\bar{r}} + R_{oe}\,\hat{e}(k_{1z}^{ed})\, e^{i\bar{K}_1{}^{e}\cdot\bar{r}}, \qquad z \leq 0$$

$$\bar{E}_o(\bar{r}) = X_{oH}\,\hat{h}_o(k_{oz})\, e^{i\bar{k}_o\cdot\bar{r}} + X_{oV}\,\hat{v}_o(k_{oz})\, e^{i\bar{k}_o\cdot\bar{r}}, \qquad z \geq 0.$$

Applying the boundary conditions, we get

$$R_{oo} = \frac{g_d}{g_u}\frac{E_e}{D_e}\frac{k_{1z}^{o} - k_{oz}}{k_{1z}^{o} + k_{oz}}$$

where

$$E_e = \cos^2\psi\, k_\rho{}^2(k_1{}^2 k_{oz} - k_o{}^2 k_{1z}^{ed}) + \sin^2\psi\{k_1{}^2(k_{oz} - k_{1z}^{ed})(k_x{}^2 - k_{oz}k_{1z}^{o})$$

$$- k_y{}^2 k_{1z}^{o}(k_1{}^2 - k_o{}^2)\} + \cos\psi \sin\psi\, k_y(k_{1z}^{o} - k_{oz})(k_{1z}^{o} + k_{1z}^{ed})(k_\rho{}^2 - k_{oz}k_{1z}^{o})$$

$$R_{oe} = \frac{U_d}{g_u}\frac{1}{D_e}\, 2k_x k_{1z}^{o}(k_{1z}^{o} - k_{oz})(k_y \sin\psi - k_{oz}\cos\psi)\sin\psi$$

$$X_{oH} = \frac{1}{k_\rho}\left\{\frac{1}{g_u}(k_\rho{}^2\cos\psi - k_y k_{1z}^{o}\sin\psi) + \frac{R_{oo}}{g_d}(k_\rho{}^2\cos\psi + k_y k_{1z}^{o}\sin\psi)\right.$$
$$\left. - \frac{R_{oe}}{U_d} k_1{}^2 k_x \sin\psi\right\}$$

$$X_{oV} = \frac{k_o}{k_{oz}k_\rho}\left\{\frac{1}{g_u}k_x k_{1z}^{o}\sin\psi - \frac{R_{oo}}{g_d}k_x k_{1z}^{o}\sin\psi - \frac{R_{oe}}{U_d}(k_y k_{1z}^{o2}\sin\psi - k_\rho{}^2 k_{1z}^{ed}\cos\psi)\right\}$$

For an incident extraordinary wave [Fig. 6b], the $\bar{E}$ fields satisfy

$$E_1(\bar{r}) = \hat{e}(k_{1z}^{eu})\, e^{i\bar{k}_1{}^e\cdot\bar{r}} + R_{eo}\,\hat{o}(-k_{1z}^{o})\, e^{i\bar{k}_1{}^o\cdot\bar{r}} + R_{ee}\,\hat{e}(k_{1z}^{ed})\, e^{i\bar{k}_1{}^e\cdot\bar{r}}, \qquad z \le 0$$

$$E_o(\bar{r}) = X_{eH}\,\hat{h}_o(k_{oz})\, e^{i\bar{k}_o\cdot\bar{r}} + X_{eV}\,\hat{v}_o(k_{oz})\, e^{i\bar{k}_o\cdot\bar{r}}, \qquad z \ge 0.$$

Applying the boundary conditions, we obtain

$$R_{ee} = -\frac{U_d}{U_u}\frac{I_e}{D_e}$$

where

$$I_e = \cos^2\psi\, k_\rho{}^2(k_1{}^2k_{oz} - k_o{}^2k_{1z}^{eu}) + \sin^2\psi\{k_1{}^2(k_{oz} - k_{1z}^{eu})(k_x{}^2 + k_{oz}k_{1z}^{o})$$

$$+ k_y{}^2k_{1z}^{o}(k_1{}^2 - k_o{}^2)\} + \cos\psi \sin\psi\, k_y(k_{1z}^{o} + k_{oz})(k_{1z}^{o} - k_{1z}^{eu})(k_\rho{}^2 + k_{oz}k_{1z}^{o})$$

$$R_{eo} = \frac{g_d}{U_u}\frac{1}{D_e}k_1{}^2k_x(k_{1z}^{ed} - k_{1z}^{eu})(k_{1z}^{o} - k_{oz})(k_y \sin\psi + k_{oz}\cos\psi)\sin\psi$$

$$X_{eH} = \frac{1}{k_\rho}\left\{-\frac{1}{U_u}k_1{}^2k_x \sin\psi + \frac{R_{eo}}{g_d}(k_\rho{}^2\cos\psi + k_yk_{1z}^{o}\sin\psi) - \frac{R_{ee}}{U_d}k_1{}^2k_x\sin\psi\right\}$$

$$X_{eV} = -\frac{k_o}{k_{oz}k_\rho}\left\{\frac{1}{U_u}(k_{1z}^{o2}k_y\sin\psi - k_\rho{}^2k_{1z}^{eu}\cos\psi) + \frac{R_{eo}}{g_d}k_xk_{1z}^{o}\sin\psi\right.$$

$$\left. + \frac{R_{ee}}{U_d}(k_yk_{1z}^{o2}\sin\psi - k_\rho{}^2k_{1z}^{ed}\cos\psi)\right\}$$

V. Conclusion

The dyadic Green's functions (DGF) for an anisotropic (tilted uniaxial) medium have been evaluated for the unbounded and the two-layered cases. Inside the uniaxial medium, there are two characteristic waves, i.e. an o-wave and an e-wave. A wave incident with either TE or TM polarization produces both TE and TM waves when it reflectes at the interface of an isotropic and a uniaxially anisotropic media because of its coupling into both the o-wave and the e-wave. With the availability of the DGF for anisotropic media, many layered-medium problems can now be solved. For instance, the problem of radiating a Hertzian dipole with an arbitrary orientation on top of a two layer anisotropic medium can readily be solved.

References

[1] C. T. Tai, Dyadic Green's Functions in Electromagnetic Theory, Intex, 1972.

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[3] J. A. Kong, Theory of Electromagnetic Waves, John Wiley and Sons, New York, pp. 59-62, 1975.

Figure Captions

Figure 1 Geometry of an anisotropic medium.

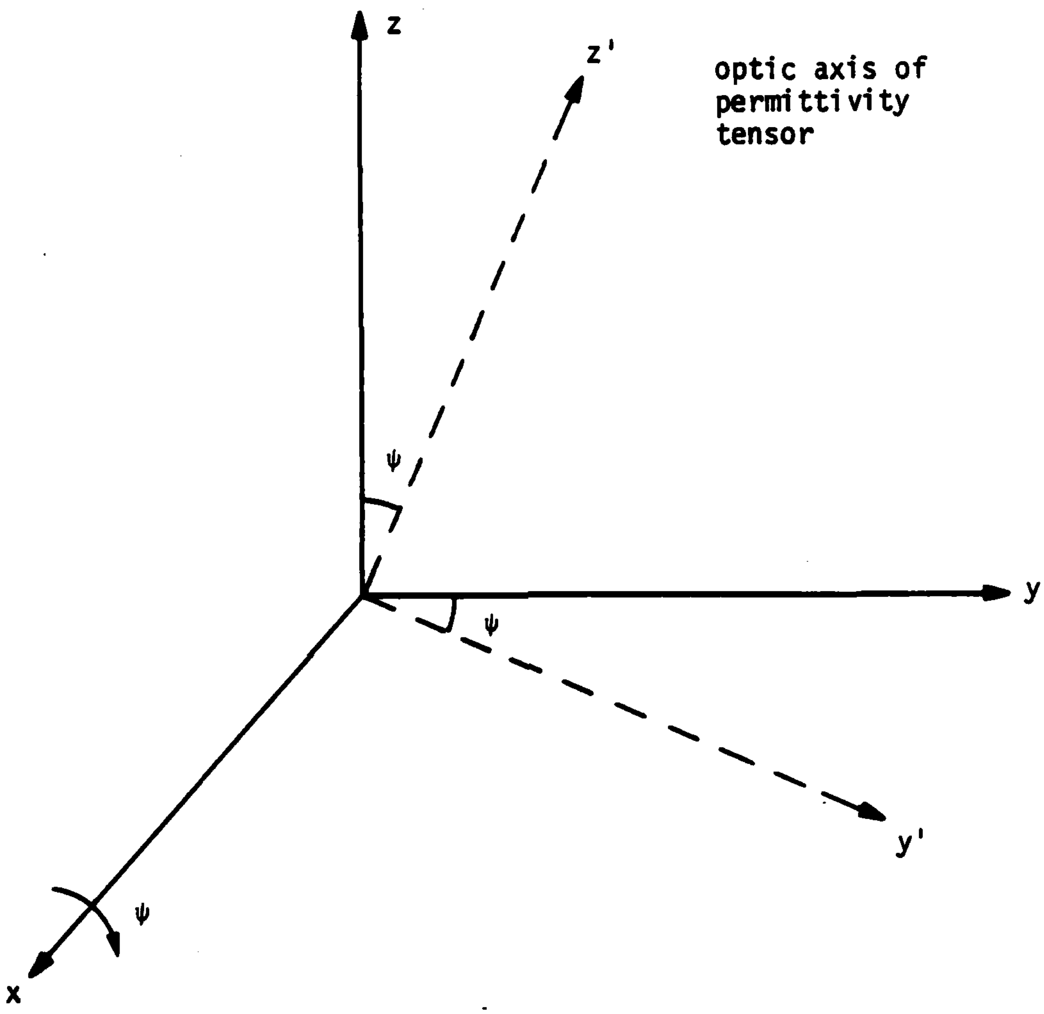
Figure 2 Geometry of a two-layered anisotropic medium.

Figure 3 Amplitude vectors of the incident, reflected and transmitted waves.

Figure 4 Reflection and transmission from an isotropic medium upon a anisotropic medium.

Figure 5 Reflection and transmission from an anisotropic medium (upper) upon an isotropic medium (lower).

Figure 6 Reflection and transmission from an anisotropic medium (lower) upon an isotropic medium (upper).



<u>Figure 1</u>

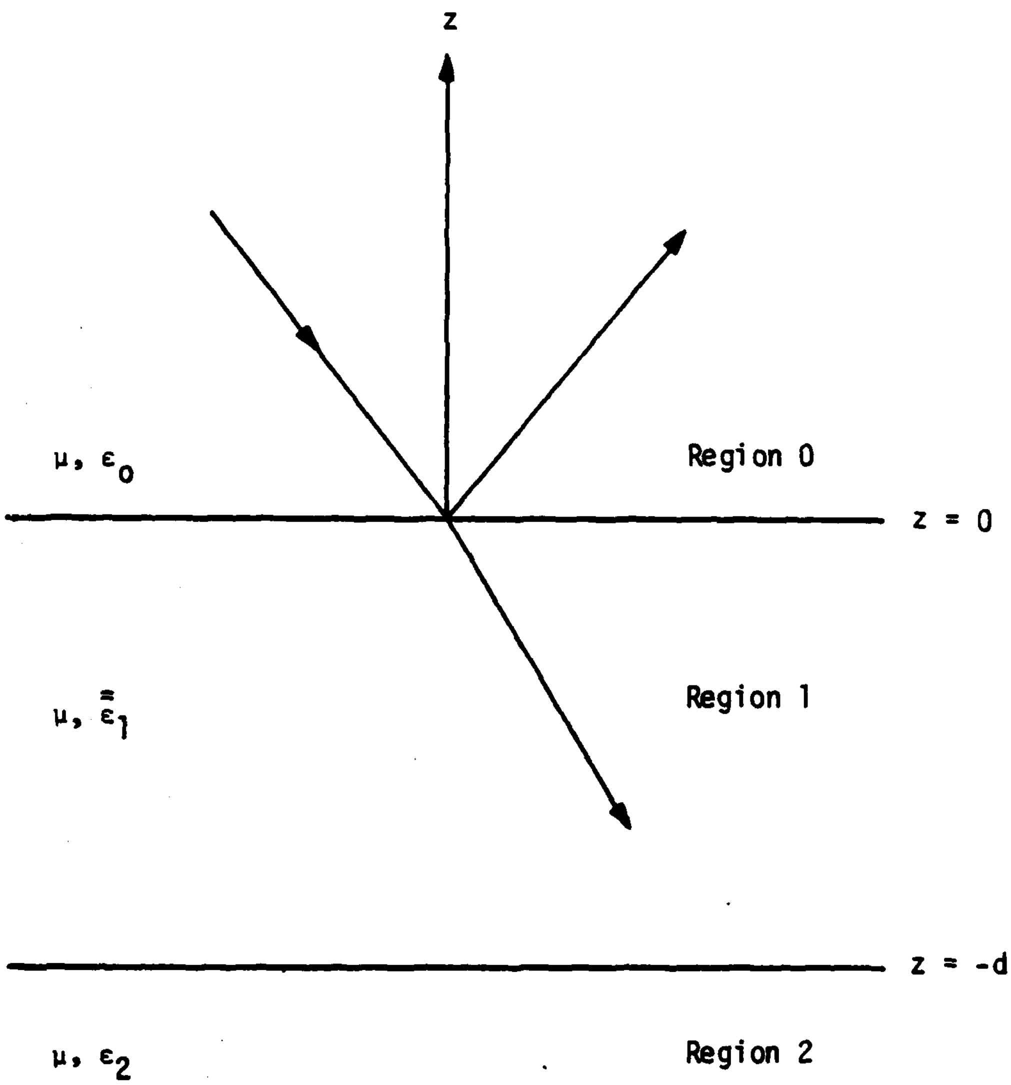


Figure 2

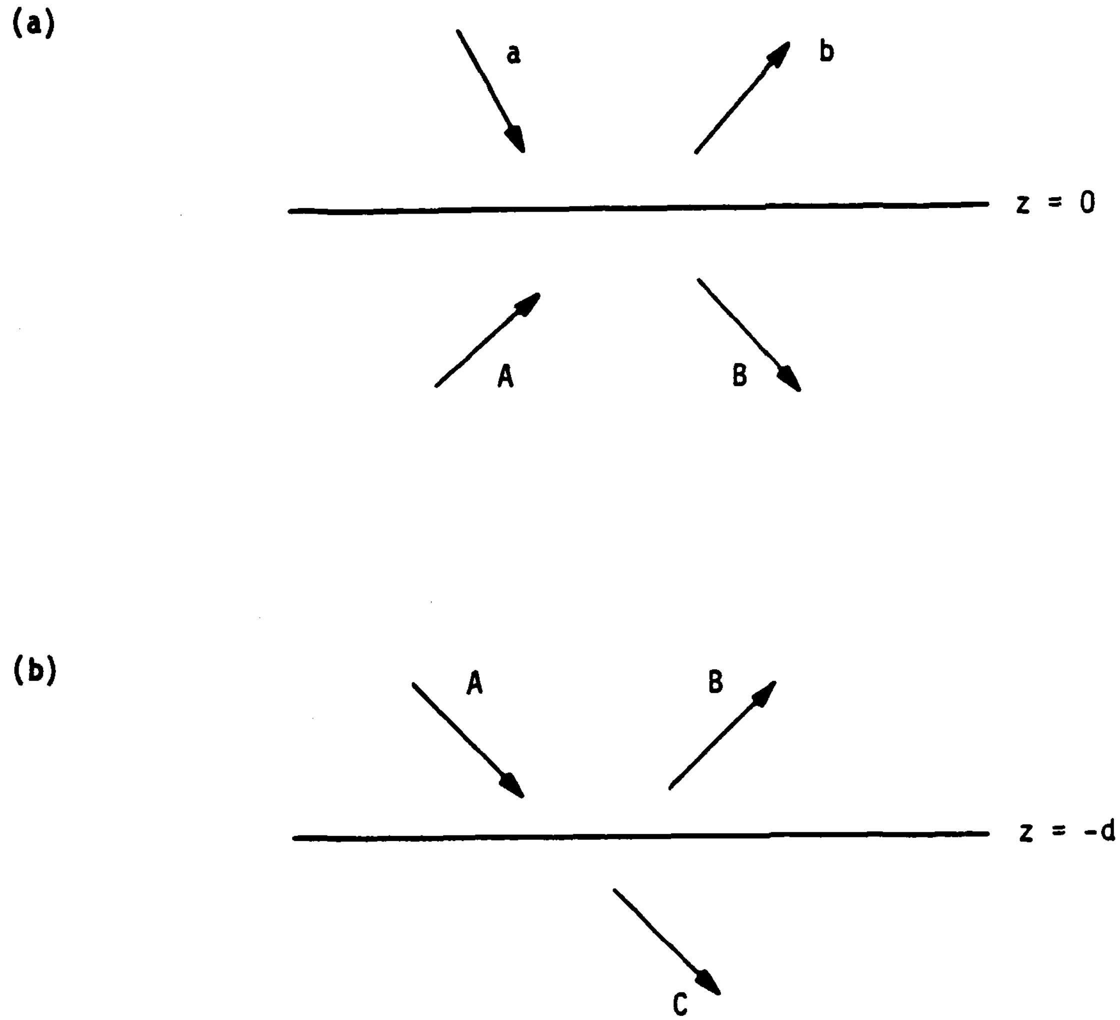


Figure 3

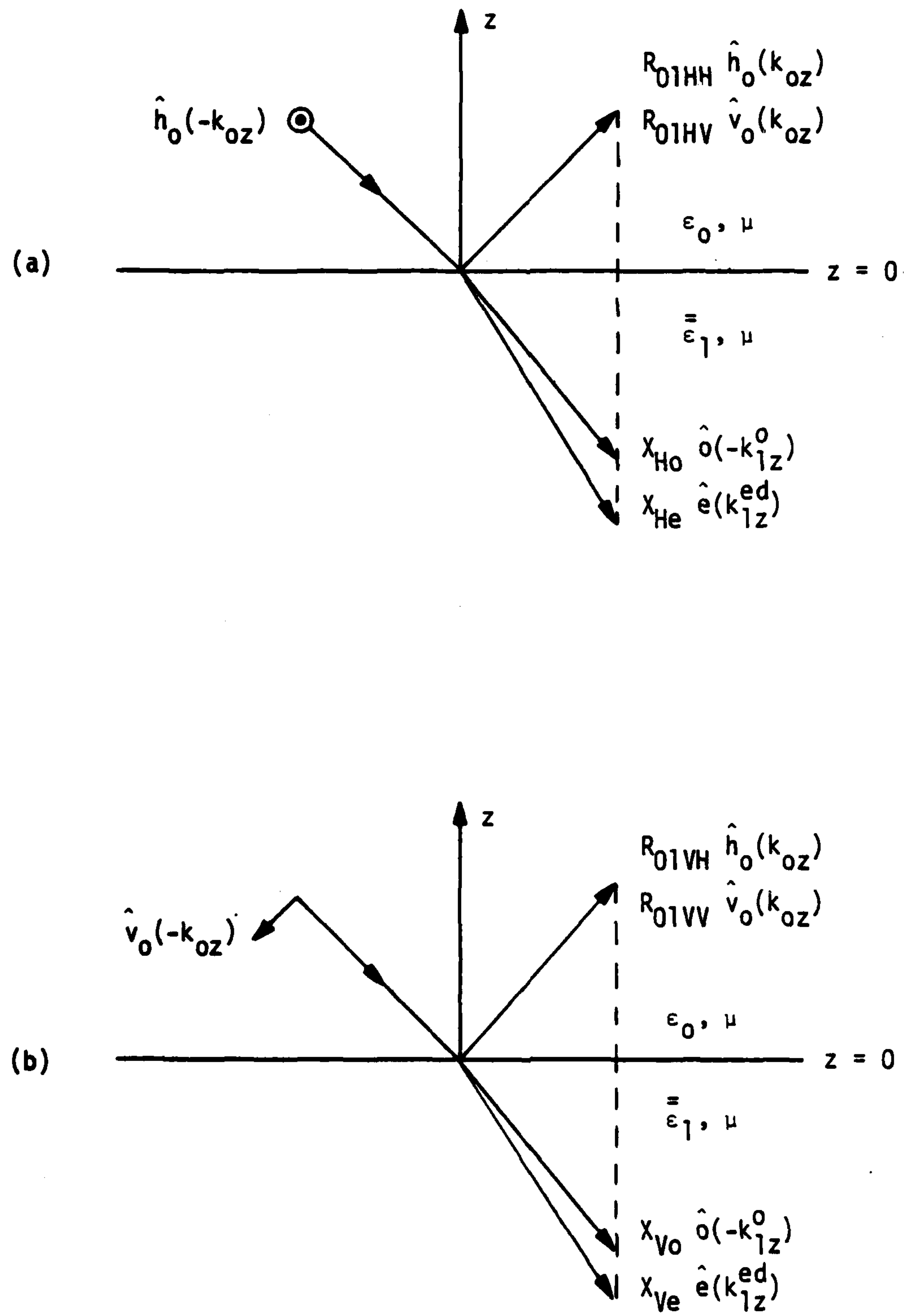
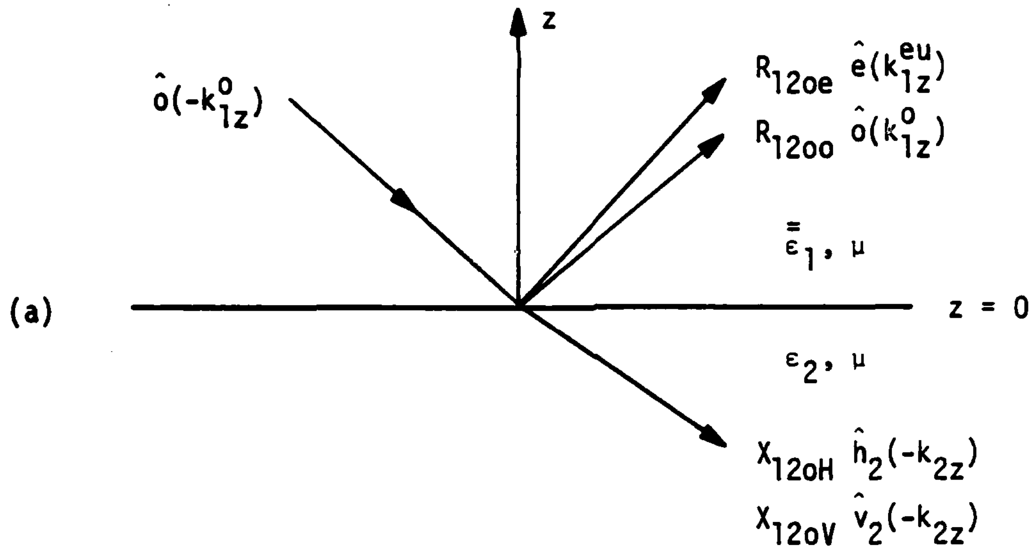

z
$R_{01HH}\ \hat{h}_o(k_{oz})$
$R_{01HV}\ \hat{v}_o(k_{oz})$
$\hat{h}_o(-k_{oz})$
$\varepsilon_o,\ \mu$
(a)
z = 0
$\bar{\bar{\varepsilon}}_1,\ \mu$
$X_{Ho}\ \hat{o}(-k^o_{1z})$
$X_{He}\ \hat{e}(k^{ed}_{1z})$
z
$R_{01VH}\ \hat{h}_o(k_{oz})$
$R_{01VV}\ \hat{v}_o(k_{oz})$
$\hat{v}_o(-k_{oz})$
$\varepsilon_o,\ \mu$
(b)
z = 0
$\bar{\bar{\varepsilon}}_1,\ \mu$
$X_{Vo}\ \hat{o}(-k^o_{1z})$
$X_{Ve}\ \hat{e}(k^{ed}_{1z})$

Figure 4


z
$\hat{o}(-k_{1z}^{o})$
$R_{12oe}\ \hat{e}(k_{1z}^{eu})$
$R_{12oo}\ \hat{o}(k_{1z}^{o})$
$\bar{\bar{\epsilon}}_1,\ \mu$
(a)
z = 0
$\epsilon_2,\ \mu$
$X_{12oH}\ \hat{h}_2(-k_{2z})$
$X_{12oV}\ \hat{v}_2(-k_{2z})$

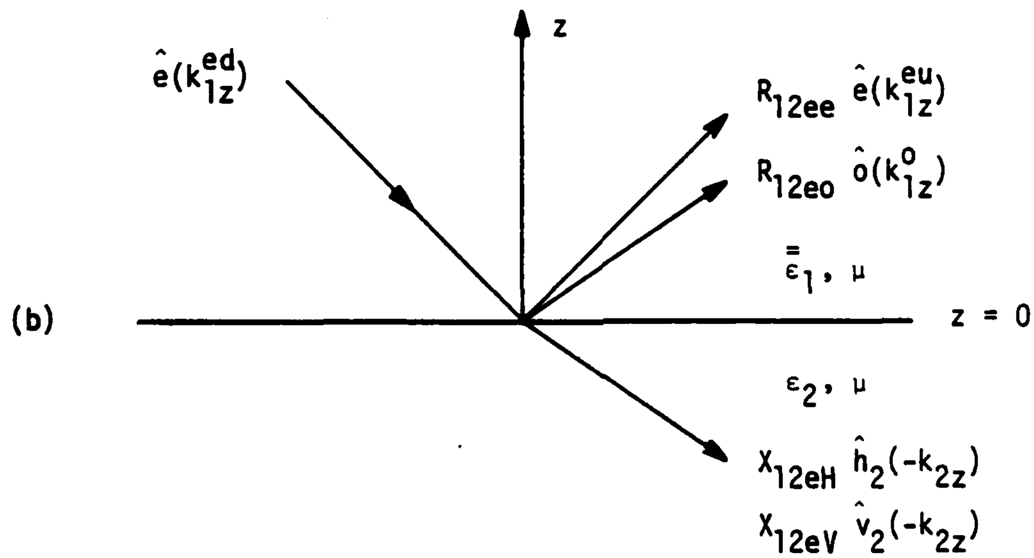
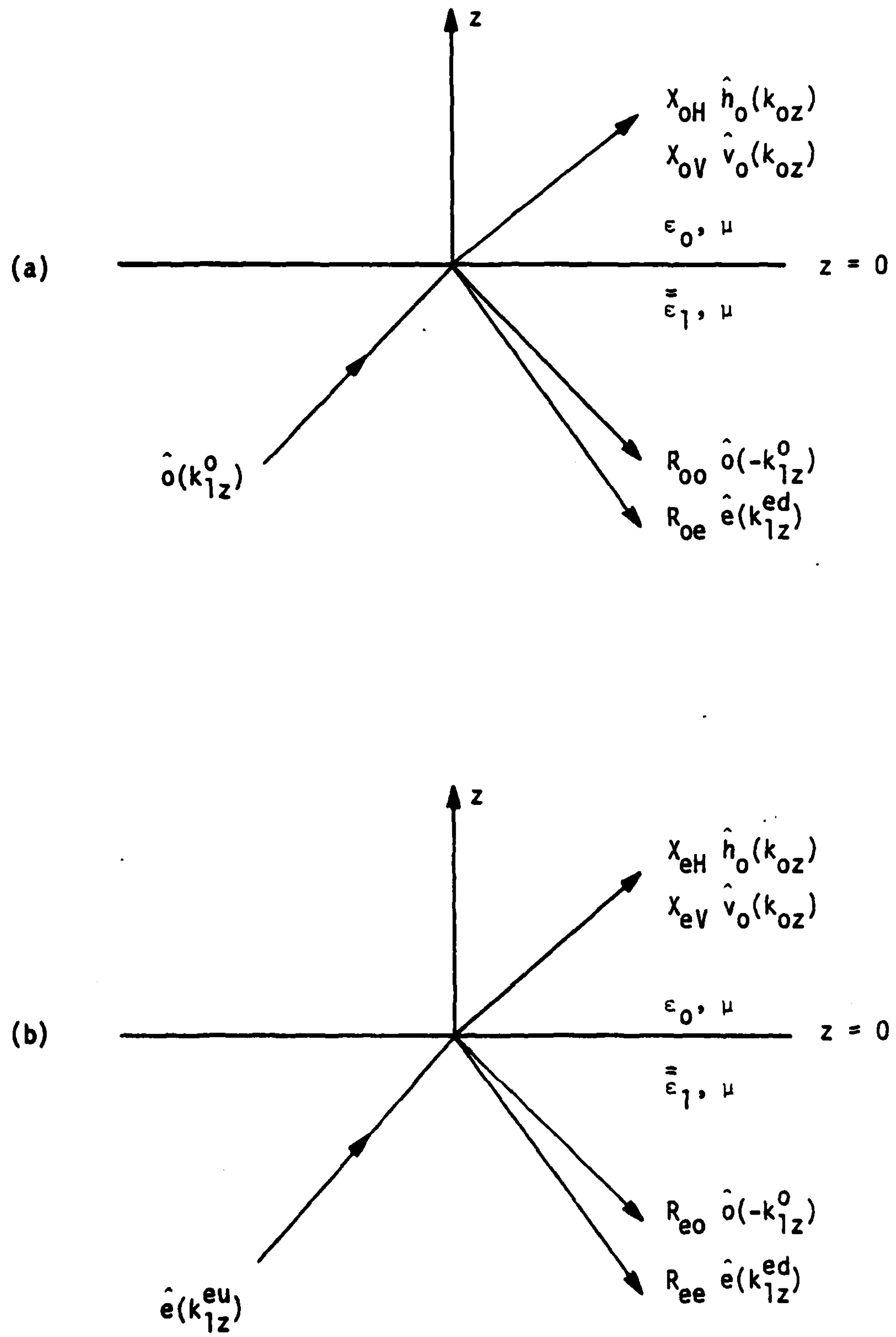

z
$\hat{e}(k_{1z}^{ed})$
$R_{12ee}\ \hat{e}(k_{1z}^{eu})$
$R_{12eo}\ \hat{o}(k_{1z}^{o})$
$\bar{\bar{\epsilon}}_1,\ \mu$
(b)
z = 0
$\epsilon_2,\ \mu$
$X_{12eH}\ \hat{h}_2(-k_{2z})$
$X_{12eV}\ \hat{v}_2(-k_{2z})$

Figure 5



(a)
z
$\chi_{oH}\ \hat{h}_o(k_{oz})$
$\chi_{oV}\ \hat{v}_o(k_{oz})$
$\varepsilon_o,\ \mu$
$z = 0$
$\bar{\bar{\varepsilon}}_1,\ \mu$
$\hat{o}(k_{1z}^{o})$
$R_{oo}\ \hat{o}(-k_{1z}^{o})$
$R_{oe}\ \hat{e}(k_{1z}^{ed})$
(b)
z
$\chi_{eH}\ \hat{h}_o(k_{oz})$
$\chi_{eV}\ \hat{v}_o(k_{oz})$
$\varepsilon_o,\ \mu$
$z = 0$
$\bar{\bar{\varepsilon}}_1,\ \mu$
$\hat{e}(k_{1z}^{eu})$
$R_{eo}\ \hat{o}(-k_{1z}^{o})$
$R_{ee}\ \hat{e}(k_{1z}^{ed})$

Figure 6

END

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